Random noise in ultrasonic echoes diffracted by blood

Random noise in ultrasonic echoes diffracted by blood

P Atkinson† and M V Berry‡

† Medical Physics, Bristol General Hospital, Guinea Street, Bristol BS1 6SY, UK
‡ H H Wills Physics Laboratory, Bristol University, Tyndall Avenue, Bristol BS8 1TL, UK

Abstract. The ultrasonic echo back-scattered by blood fluctuates as a function of time delay and lateral displacement of the source–receiver. This granular echo is not due to any special structure in the blood on the scale observed, but probably arises from fluctuation scattering by the random distribution of red cells; the dimensions of the ultrasonic pulse determine the scale of fluctuation detected. A statistical diffraction theory is developed, and formulae derived for the mean relative echo envelope Π, and the mean rates of fluctuation N_t and N_R of the envelope about its mean as the time delay and lateral displacement are varied. N_t and N_R agree reasonably with experiment, but Π is in error by a factor of thirteen; this discrepancy is discussed, and possible explanations suggested.

1. Introduction

A piezoelectric crystal, acting as a transmitter and receiver of ultrasonic waves, was immersed in a vessel containing blood, and quasi-monochromatic pulses emitted, each consisting of a small number of waves corresponding to a predominant frequency $\omega/2\pi$ of 2 MHz (figure 1(a)). The weak echo returning from the bulk liquid, when displayed as a curve of sound pressure P against time t, showed a division of the waves into quasi-random 'groups' (figure 1(b)); the duration of each group was typically about 3–4 cycles, corresponding to a spatial extent of about 2·5 mm. The *envelope* of the echo was also examined, by electronically rectifying the sound pressure signal and smoothing over a time comparable with the wave period $2\pi/\omega$; the resulting positive 'demodulated' quantity, which we call P_{env}, also showed a division into groups (figure 1(c)). Next the crystal was moved in a plane parallel to its transmitting face, to explore different columns of blood; the *spatial* variation of P_{env} was recorded as a function of a two-dimensional displacement vector R_0 at fixed t. Again quasi-random behaviour was observed (figure 1(d)), with variation on a scale of about 10 mm.

At first these results suggested that blood has a granular structure on the scale observed; however, no such structure seems to have been seen in a microscope, or detected by any other method (of course electromagnetic structure, as shown by a microscope, need not be the same as acoustical structure). The only significant known structure is on a much smaller scale: red blood cells (that is, corpuscles), in the form of biconcave discs several microns across, are densely distributed throughout the plasma (the white corpuscles and 'platelets' are negligible). The pulse echo technique could not possibly resolve individual corpuscles, because the sound wavelength λ is much too large (if v is the speed of sound in blood, $\lambda = 2\pi v/\omega = 0.75$ mm). However, if the blood cells are randomly distributed (as seems to be the case), then this implies that the numbers contained in different small volumes of the same size V will not simply be given by nV,
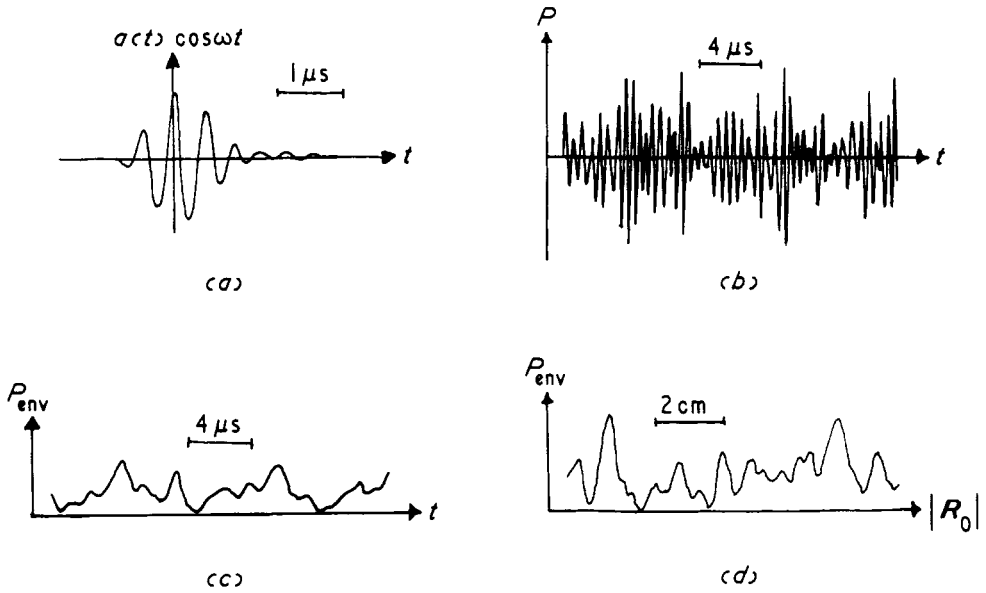
Figure 1. (a) Time dependence of pressure P in incident wave; (b) time dependence of pressure P in back-scattered echo; (c) time dependence of echo envelope P_{env}; (d) dependence of P_{env} on lateral displacement of source–receiver. (In (b), (c) and (d) the vertical scales are approximately the same.)

where n is the overall number density, but will *fluctuate* about this value. In particular, if the linear dimensions of V are taken approximately equal to the ultrasonic wavelength, or the pulse length, or the transverse dimensions of the beam, these fluctuations imply variations in scattering power throughout the specimen†. This suggests that the granular structure of the echo as a function of t and R_0 is a *random noise effect*, in which the dimensions of the ultrasonic pulse determine the scale of fluctuation detected. By varying the pulse length we obtain immediate support for this idea: it is found that the average interval between envelope maxima (figure 1(c)) varies in direct proportion to the pulse length. This effect cannot be a consequence of smoothing in the receiving electronics being less effective for short pulses (where envelope maxima might be confused with carrier-wave maxima), because for all our pulses the ratio of $\omega/2\pi$ to bandwidth exceeded 5.

In the rest of this paper we examine further consequences of assuming that the echo results from fluctuation scattering, analogous to the Rayleigh–Tyndall scattering responsible for the blue of the sky (Van de Hulst 1957). Section 2 is theoretical; we calculate the mean value $\langle P_{env} \rangle$ of the echo envelope returned from an assembly of corpuscles, and predict the 'fading rate', that is, the rate at which P_{env} fluctuates about $\langle P_{env} \rangle$ as t and R_0 are varied. This theory is applicable not just to diffraction of ultrasound by blood, but describes fluctuations in the echo from any system of scatterers which are small in comparison with the predominant wavelength of the incident radiation. In § 3 we compare the results with experiment. A detailed description of the apparatus and procedure, and an account of the clinical context of the experiments, will be given in a later publication by one of us (PA).

† These variations could not arise from fluctuations in the *orientation* of cells in a uniform distribution, because for such small scatterers the diffraction pattern is independent of their shape, and hence of their orientation.

First, however, we introduce and eliminate a red herring. The red corpuscles will execute a brownian motion as a result of impacts from molecules in the plasma. This will cause the fluctuations in scattering power continually to dissolve and re-appear at different places. Thus echo patterns such as those of figures 1(*b, c* and *d*) should gradually change. Such 'secular' changes do actually occur, over times of several minutes, but they do *not* arise from brownian motion, for a least two reasons. The first is theoretical: the 'dissolving time' t_d of an observable fluctuation cannot be less than the time taken for a corpuscle to wander a net distance of the order of the wavelength λ in its 'random walk' through the blood plasma. Standard theory (Reif 1965, p 567) gives

$$t_d > \frac{3\pi\eta a\lambda^2}{kT},\tag{1}$$

where η is the shear viscosity of plasma, a is a typical dimension of a red corpuscle, T the absolute temperature and k is Boltzmann's constant. Substitution of numerical values into equation (1) gives $t_d > 10^8$ s, so that brownian motion is far too slow to account for the observed changes in the pattern of echo fluctuations. The second reason is experimental: over several hours, these changes in the echo gradually slow down; if brownian motion were responsible, the changes would persist forever. But why does the echo change with time? We conjecture that the changes are due to *convection*, which carries regions of varying scattering power through the volume being interrogated by the pulse. Over a period of several hours the convection currents gradually die away; eventually the red blood cells settle out of the plasma, and the system loses its bulk homogeneity.

2. Statistical theory of diffraction by small particles

To describe the sound wave we use the acoustic pressure P. This is obviously a real function of position and time, but the diffraction theory is more easily formulated in terms of a *complex* function P_c, whose real part is P. Both P_c and P are quasimonochromatic, and it is easily shown (using, for example, the method of Berry 1973, § 2(i)) that the envelope function P_{env} is given by

$$P_{env} = |P_c|\tag{2}$$

(a relation which always holds, irrespective of whether or not scattering has occurred). In the blood we set up coordinates Z perpendicular to the transmitting face of the crystal and $\boldsymbol{R}$ ($= x, y$) parallel to this face. Then the pressure $P_c^{inc}(\boldsymbol{R}, Z, t)$ in the incident wave (that is, before scattering) depends on the time dependence of the pulse emerging from the crystal, which can be described by the dimensionless function $a(t)\exp(-i\omega t)$, and also on the shape of the wave produced by diffraction from the crystal. It turns out that the side lobes of the diffraction pattern can be neglected and the central lobe approximated by a plane wave transversely modulated by an amplitude which can be described by the cylindrically symmetrical dimensionless function $b(R)$, at least in the region we are interested in. Thus we take

$$P_c^{inc}(\boldsymbol{R}, Z, .) = P_0 a[t - (Z/v)] \exp[i(kZ - \omega t)] b(|\boldsymbol{R} - \boldsymbol{R}_0|),\tag{3}$$

where $k \equiv 2\pi/\lambda = \omega/v$, P_0 is a constant pressure amplitude which depends on the energy fed into the crystal, and $(\boldsymbol{R}_0, 0)$ is the position of the transducer (figure 2). We
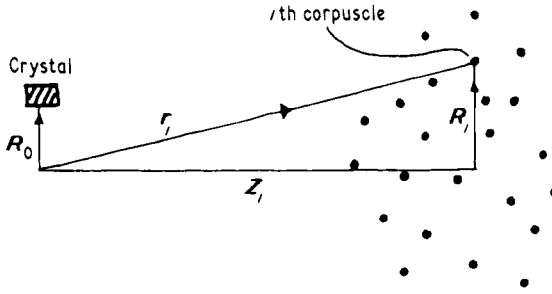
Figure 2. Coordinates of crystal and scatterers. The transmitting face of the crystal is perpendicular to the Z axis.

see from figure 1(*a*) that the pulse envelope $a(t)$ is approximately gaussian, and we shall see in § 3 that the beam-width function $b(R)$ falls off approximately exponentially.

As P_c^{inc} strikes each red cell at its position r_i ($= R_i Z_i$, figure 2), a scattered wave is excited, whose amplitude falls off linearly with distance. Eventually all these scattered waves reach the receiver; provided no multiple scattering occurs (see § 3) these add up to give an echo $P_c(t, R_0)$ of the form

$$P_c(t, R_0) = P_0 A \, e^{-i\omega t} \sum_i a\!\left(t - \frac{2Z_i}{v}\right) b(|R_i - R_0|) \frac{\exp(2ikZ_i)}{Z_i} \tag{4}$$

where the summation is over all the corpuscles, and we have neglected powers of $|R_i - R_0|/Z_i$ higher than the first (an approximation amply justified in the present case). A is the amplitude for backward scattering from a flexible object minute in comparison with the wavelength; this is shown by Rschevkin (1963) to be

$$A = \frac{\tau k^2}{4\pi}\left(\frac{\bar{\chi} - \chi}{\bar{\chi}} + \frac{3(\bar{\rho} - \rho)}{2\bar{\rho} + \rho}\right), \tag{5}$$

where τ is the (average) volume of a single corpuscle, and $\bar{\chi}$, $\bar{\rho}$ and χ, ρ are the adiabatic bulk moduli and mass densities of red cells and blood plasma respectively. (Strictly speaking, k^2 in equation (5) should be a mean value, since the pulse is not mono-chromatic, but no appreciable error arises from using ω^2/v^2, since all frequencies in the pulse correspond to wavelengths much greater than the size of the scatterers.) If we define $t = 0$ as the instant that the maximum of the pulse is emitted, we can use the fact that $a(t)$ is a localized function to replace Z_i by its approximate value $vt/2$, to get the final echo formula

$$P_c(t, R_0) = \frac{2P_0 A \, e^{-i\omega t}}{vt} \sum_i a\!\left(t - \frac{2Z_i}{v}\right) b(|R_i - R_0|) \exp(2ikZ_i). \tag{6}$$

Now we must calculate the mean value $\langle P_{env}\rangle$, and the fluctuations of P_{env} about the mean. It is simplest to calculate the *mean square modulus* $\langle |P_c|^2\rangle$ of the echo, and use the relation

$$\langle P_{env}\rangle = \tfrac{1}{2}\sqrt{\pi}\langle |P_c|^2\rangle^{1/2}, \tag{7}$$

which follows from equation (2) together with the fact that $|P_c|$ possesses a Rayleigh distribution because P_c is a gaussian random variable (we shall justify this statement in the paragraph following equation (16)). Using equation (6) we obtain, for the mean

square echo modulus at a given time t,

$$\langle |P_c|^2 \rangle = \left(\frac{2P_0 A}{vt} \right)^2 \sum_i \sum_j \left\langle a\left(t - \frac{2Z_i}{v} \right) a\left(t - \frac{2Z_j}{v} \right) \exp[2ik(Z_i - Z_j)] \right.$$

$$\left. \times b(|\boldsymbol{R}_i - \boldsymbol{R}_0| b(|\boldsymbol{R}_j - \boldsymbol{R}_0|) \right\rangle. \tag{8}$$

The brackets $\langle \dots \rangle$ denote an ensemble average over all possible positions of the scatterers. Let there be $\mathcal{N}$ red cells in the vessel, which has a volume Ω large in comparison with the travelling pulse of ultrasound. Of the $\mathcal{N}^2$ terms in the summations in equation (8), $\mathcal{N}$ have $i = j$, while $\mathcal{N}(\mathcal{N} - 1)$ have $i \neq j$. Thus we require both the *singlet probability distribution*, given by $P_i(\boldsymbol{r}_i) \, \mathrm{d}\boldsymbol{r}_i$, the probability that the ith corpuscle lies in a small volume $\mathrm{d}\boldsymbol{r}_i$ centred on $\boldsymbol{r}_i$:

$$P_i(\boldsymbol{r}_i) \, \mathrm{d}\boldsymbol{r}_i = \frac{\mathrm{d}\boldsymbol{r}_i}{\Omega}, \tag{9}$$

and also the *doublet probability distribution*, given by $P_2(\boldsymbol{r}_i, \boldsymbol{r}_j) \, \mathrm{d}\boldsymbol{r}_i \, \mathrm{d}\boldsymbol{r}_j$, the probability that the ith corpuscle lies in a small volume $\mathrm{d}\boldsymbol{r}_i$ centred on $\boldsymbol{r}_i$ *and* the jth corpuscle lies in a small volume $\mathrm{d}\boldsymbol{r}_j$ centred on $\boldsymbol{r}_j$:

$$P_2(\boldsymbol{r}_i, \boldsymbol{r}_j) \, \mathrm{d}\boldsymbol{r}_i \, \mathrm{d}\boldsymbol{r}_j = \frac{\mathrm{d}\boldsymbol{r}_i \, \mathrm{d}\boldsymbol{r}_j}{\Omega^2} g(\boldsymbol{r}_i - \boldsymbol{r}_j), \tag{10}$$

where $g(\boldsymbol{r}_i - \boldsymbol{r}_j)$ is the pair correlation function.

When i and j are well separated, $g(\boldsymbol{r}_i - \boldsymbol{r}_j)$ is unity, since the corpuscles are statistically independent, while the impossibility of interpenetration of two corpuscles means that $g(\boldsymbol{r}_i - \boldsymbol{r}_j)$ is zero if $|\boldsymbol{r}_i - \boldsymbol{r}_j|$ is less than a few microns. Now the fastest variation in the function to be averaged in equation (8) occurs over distances of order λ, while $g(\boldsymbol{r}_i - \boldsymbol{r}_j)$ differs from unity over regions a thousand times smaller than λ; thus to an excellent approximation we can write

$$g(\boldsymbol{r}_i - \boldsymbol{r}_j) = 1 - \delta(\boldsymbol{r}_i - \boldsymbol{r}_j) \int \int \int \mathrm{d}\boldsymbol{r}(1 - g(r)) \equiv 1 - \alpha\delta(\boldsymbol{r}_i - \boldsymbol{r}_j), \tag{11}$$

where $\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)$ is the Dirac delta function and α is a measure of the 'excluded volume' surrounding a corpuscle, which we shall identify more precisely in a moment.

It is now possible to work out the averages in equation (8), and we obtain

$$\langle |P_0|^2 \rangle = \left(\frac{2P_0 A}{vt} \right)^2 \left[\left(\frac{\mathcal{N}}{\Omega} - \frac{\alpha \mathcal{N}(\mathcal{N} - 1)}{\Omega^2} \right) \int \int \mathrm{d}\boldsymbol{R} b^2(R) \int_{-\infty}^{\infty} \mathrm{d}Z a^2\left(t - \frac{2Z}{v} \right) \right.$$

$$\left. + \frac{\mathcal{N}(\mathcal{N} - 1)}{\Omega^2} \left| \int \int \mathrm{d}\boldsymbol{R} b(R) \int_{-\infty}^{\infty} \mathrm{d}Z \, \mathrm{e}^{2ikZ} a\left(t - \frac{2Z}{v} \right) \right|^2 \right]. \tag{12}$$

The integral involving the oscillatory factor $\exp(2ikZ)$ is completely negligible (in fact we can show that its value is of order $\exp(-4\pi^2 m^2)$, where m is the number of waves in the pulse). Only the integrals without oscillatory factors are significant, and if we define $n = \mathcal{N}/\Omega$ as the number density of corpuscles, we obtain, for the mean square echo modulus,

$$\langle |P_c|^2 \rangle = \frac{4\pi}{v} \left(\frac{P_0 A}{t} \right)^2 n(1 - \alpha n) \int_{-\infty}^{\infty} \mathrm{d}t \, a^2(t) \int_0^{\infty} \mathrm{d}R \, R b^2(R). \tag{13}$$

($\mathcal{N} - 1$ has here been replaced by $\mathcal{N}$; this is certainly permissible since the number of scatterers in the vessel is effectively infinite.) When αn is unity, the reflected intensity given by this expression is zero. But we are dealing with fluctuation scattering, which can only vanish for a homogeneous medium. Thus αn would be unity only when the whole volume of 'blood' was packed full with corpuscles. This means that we can identify α with the volume τ of an average corpuscle, at least for dense distributions.

The mean value $\langle P_{env} \rangle$ of the pressure envelope follows directly from equations (13) and (7). Experimentally we compare $\langle P_{env} \rangle$ with the peak envelope P_{env}^{ref} received from a perfect reflector placed at $Z = vt/2$. This can easily be calculated from equations (2) and (3), and the resulting ratio Π is

$$\Pi \equiv \frac{\langle P_{env} \rangle}{P_{env}^{ref}} = \frac{\pi A}{t} \left[\frac{n(1 - \alpha n)}{v} \int_{-\infty}^{\infty} dt \left| \frac{a(t)}{a(0)} \right|^2 \int_0^{\infty} dR\, R \left| \frac{b(R)}{b(0)} \right|^2 \right]^{1/2}. \tag{14}$$

We shall see in § 3 that $a(t)$ and $b(R)$ can be well represented by

$$a(t) = \exp(-t^2/2T^2) \tag{15a}$$

$$b(R) = \exp(-R/R_B), \tag{15b}$$

where T is a measure of the pulse length and R_B a measure of the beam radius at $Z \sim vt/2$. Thus Π becomes, finally,

$$\Pi = \frac{\pi A R_B}{2t} \left(\frac{\pi^{1/2} n(1 - \alpha n)T}{v} \right)^{1/2}, \tag{16}$$

where A is given by equation (5).

As t and $\boldsymbol{R}_0$ vary, the echo envelope P_{env} fluctuates about its mean value $\langle P_{env} \rangle$ given by equations (13) and (7); these fluctuations can be seen in figures 1(c and d). The simplest quantities characterizing the fluctuations are two 'fading rates' N_t and N_R, defined as follows: N_t is the mean number of crossings of P_{env} through the value $\langle P_{env} \rangle$ during unit interval of t with the crystal position $\boldsymbol{R}_0$ held fixed. N_R is the mean number of crossings of P_{env} through the value $\langle P_{env} \rangle$ during a unit transverse displacement of the crystal along a straight line with t held fixed.

To calculate N_t and N_R we need to know the statistical distribution of the echo $P_c(t, \boldsymbol{R}_0)$. In the present case this is *gaussian random*, because the echo (equation (6)) is the sum of a great number of statistically independent contributions. This number is roughly equal to the number of red blood cells in the volume instantaneously occupied by the sound pulse, that is, roughly $\pi n R_B^2 vT \sim 5 \times 10^7$. (This number would still be large, even if a significant amount of aggregation occurred—but see § 4.) We therefore apply standard gaussian noise theory (Rice 1944, 1945, Longuet-Higgins 1956, summarized by Berry 1973) to the function $P_c(t, \boldsymbol{R}_0)$. This theory shows that the fading rates N_t and N_R, which refer to the *envelope*, may be calculated from the autocorrelation function $C(t, \boldsymbol{R})$ of the *complex echo pressure* $P_c \exp(i\omega t)$ (the exponential factor removes the non-random time factor in equation (6)). This autocorrelation function is defined as

$$C(t, \boldsymbol{R}) \equiv \frac{\langle e^{i\omega t} P_c(t_1 + t, \boldsymbol{R}_0 + \boldsymbol{R}) P_c^*(t_1, \boldsymbol{R}_0) \rangle - |\langle P_c \rangle|^2}{\langle |P_c|^2 \rangle - |\langle P_c \rangle|^2}. \tag{17}$$

Then N_t and N_R are given (cf equations (4.38), (4.27) and (4.19) of Berry 1973) by

$$N_t = e^{-\pi/4}\left(-\frac{\partial^2 C(0,0)}{\partial t^2}\right)^{1/2} \tag{18a}$$

$$N_R = e^{-\pi/4}\left(-\frac{\partial^2 C(0,0)}{\partial|R|^2}\right)^{1/2}. \tag{18b}$$

The procedure for calculating the averages in equation (17) is precisely similar to that used to derive equation (13) from equation (8), and we obtain

$$C(t,R) = \frac{\int_{-\infty}^{\infty} dt_1 a(t_1+t)a(t_1)\int\int dR_0\, b(|R_0+R|)b(R_0)}{\int_{-\infty}^{\infty} dt\, a^2(t)\int\int dR_0\, b^2(R_0)}. \tag{19}$$

Tedious but elementary analysis gives

$$\frac{\partial^2 C(0,0)}{\partial t^2} = -\frac{\int_{-\infty}^{\infty} dt[da(t)/dt]^2}{\int_{-\infty}^{\infty} dt\, a^2(t)} \tag{20a}$$

$$\frac{\partial^2 C(0,0)}{\partial|R|^2} = -\frac{\frac{1}{2}\int_0^{\infty} dR\, R[db(R)/dR]^2}{\int_0^{\infty} dR\, Rb^2(R)}. \tag{20b}$$

Thus the fading rates may be calculated from equations (18), provided the pulse envelope $a(t)$ and the beam-width function $b(R)$ are known. In particular, if the simple forms given by equations (15) are valid, we have

$$N_t = \frac{e^{-\pi/4}}{T\sqrt{2}} = \frac{0\cdot322}{T} \tag{21a}$$

$$N_R = \frac{e^{-\pi/4}}{R_B\sqrt{2}} = \frac{0\cdot322}{R_B}. \tag{21b}$$

3. Comparison with experiment

The formulae (21) for the fading rates N_t and N_R involve only the quantities T and R_B characterizing the sound beam; the only role played by the blood is to provide a continuous spectrum of fluctuations in scattering power, producing random noise in the diffracted ultrasound. On the other hand, the formula (16) for the mean reflected relative envelope Π does involve properties of the blood: n, α, v, ρ, $\bar{\rho}$, χ, $\bar{\chi}$ and τ.

Figure 1(a) shows that the pulse envelope $a(t)$ is slightly asymmetrical; however, no serious error will be made by employing equation (15a), if we use the fact that the standard deviation T is the full width at half maximum of the gaussian curve, divided by $(8\ln 2)^{1/2} = 2\cdot355$, and realize that the full width of $a(t)$ is easily measured. This leads to the value $T = 0\cdot45\pm0\cdot03\,\mu s$. From equation (21$a$) we can calculate N_t, and this quantity can also be measured directly from curves like that shown on figure 1(c). We obtain

$$N_t = \begin{cases} (0\cdot72\pm0\cdot05)\,\text{MHz} & \text{(theory)} \\ (0\cdot53\pm0\cdot05)\,\text{MHz} & \text{(experiment)} \end{cases} \tag{22}$$

That the beam-width function $b(R)$ is approximately exponential is shown by figure 3, which is a plot of the logarithm of b^2 against R. From equation (14b), R_B is
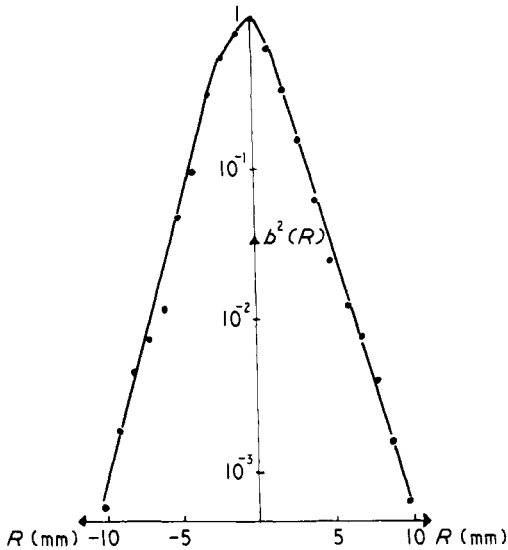
Figure 3. Logarithmic plot of measurements of square of beam-width function b^2 against radial distance R.

simply equal to twice the inverse slope of the best straight-line fit to figure 3. We obtain the value $R_B = (2.5 \pm 0.4)$ mm. From equation (21b) we can calculate N_R, and this quantity can also be measured directly from curves like that shown in figure 1(d). We obtain

$$N_R = \begin{cases} (130 \pm 20) \text{ m}^{-1} & \text{(theory)} \\ (140 \pm 20) \text{ m}^{-1} & \text{(experiment).} \end{cases} \tag{23}$$

Turning now to the calculation of the mean relative echo envelope Π, we see from equation (16) that the back-scattering amplitude A is required. This is given by equation (5). For the average volume of a corpuscle we take $\tau = (90 \pm 10)\,\mu\text{m}^3$ (Keele and Neil 1971) while the wavenumber is $k = 2\pi/\lambda = 8.4$ mm^{-1}. The ratio of densities of red blood cells and plasma is easily measured to be $\bar{\rho}/\rho = 1.1$. The ratio $\bar{\chi}/\chi$ of bulk moduli is given in terms of the sound velocities $\bar{v}$ and v (in red cell material and pure plasma respectively) by

$$\frac{\bar{\chi}}{\chi} = \frac{\bar{\rho}}{\rho}\left(\frac{\bar{v}}{v}\right)^2. \tag{24}$$

We have measured $\bar{v}$ and v, and find them identical (1500 ms^{-1}) within experimental error ($\pm 7\%$). Thus A can be calculated, and we obtain

$$A = (9 \pm 1) \times 10^{-11} \text{ m.} \tag{25}$$

To calculate Π from equation (16) we also require the corpuscle number density n; according to Keele and Neil (1971), this is 5×10^{15} m^{-3}. The volume fraction occupied by scatterers is thus $\tau n = 0.45 \simeq \alpha n$ (cf equation (11) and the remarks following equation (13)). For the time delay t at which the average envelope is measured, we choose $t = 100\,\mu\text{s}$, corresponding to a range of $vt/2 = 75$ mm. The experimental value of Π must be corrected for the finite radius R_T of the face of the crystal transducer, which

means that instead of the peak value $P_{\text{env}}^{\text{ref}}$ received from a perfect reflector, a smaller *average* value $(P_{\text{env}}^{\text{ref}})_{\text{av}}$ will in fact be measured. This is given by

$$(P_{\text{env}}^{\text{ref}})_{\text{av}} = \frac{\int\int_{R < R_T} \mathrm{d}\boldsymbol{R}\, b(R)}{\pi R_T^2 b(0)} P_{\text{env}}^{\text{ref}} = \frac{2R_B^2}{R_T^2}\left[1 - \left(1 + \frac{R_T}{R_B}\right) e^{-R_T/R_B}\right] P_{\text{env}}^{\text{ref}}, \quad (26)$$

if equation (15*b*) is used. The value of R_T is 7·5 mm, so that $(P_{\text{env}}^{\text{ref}})_{\text{av}} = 0\cdot175\, P_{\text{env}}^{\text{ref}}$. After making this correction, we finally obtain

$$\Pi = \begin{cases} (4\cdot4 \pm 0\cdot8) \times 10^{-6} & \text{(theory)} \\ (5\cdot9 \pm 0\cdot2) \times 10^{-5} & \text{(experiment)}. \end{cases} \quad (27)$$

The smallness of these quantities, and of A (equation (25)), in comparison with the distance between corpuscles, justifies our neglect of multiple scattering in writing down the basic diffraction equation (4).

4. Discussion

Equation (22) shows that the time fading rate N_t, as calculated from equation (21*a*), is of the same order of magnitude as the observed value, but the two numbers do not agree within experimental error. Equation (23) shows that the fading rate N_R for lateral motion, as calculated from equation (21*b*), does agree with experiment. These results suggest that the echo does indeed have the gaussian noise character that we have assumed.

However, equation (27) shows that the measured value of the relative average pressure envelope Π exceeds that predicted from equation (16) by a factor of thirteen. We do not believe that this discrepancy invalidates the proposed mechanism for echo formation (fluctuation scattering from red blood cells), for three reasons: (i) any tendency of the red cells to form aggregations has been ignored; the arguments of § 3 show that if a fraction q of the corpuscles aggregates into 'rouleaux' of m corpuscles each, the value of Π will increase by a factor $1 + (mq)^{1/2}$. (It is thought, however, that such *rouleaux* will not commonly occur in healthy blood (Dacie and Lewis 1970).) (ii) Equations (16) and (5) may be cast into the form of products of dimensionless ratios of the lengths λ, R_B, vT, vt, $n^{-1/3}$ and $\tau^{1/3}$. These various ratios vary by up to four orders of magnitude, so that the agreement of equations (27) within one order of magnitude is not likely to be coincidental. (iii) Although our theory of diffraction from the blood is precise within the assumptions made, in writing equation (3) we have treated in a very cavalier fashion the diffraction effects resulting from the size of the transducer. This will certainly introduce errors into the interpretation of the echo $P_{\text{env}}^{\text{ref}}$ received from the perfect reflector. Unfortunately, the region of the blood most conveniently interrogated ($t = 100\,\mu\text{s}$) lies in the transition region between Fresnel and Fraunhofer diffraction, which cannot be treated in a simple manner.

Acknowledgments

References

Berry M V 1973 *Phil. Trans.* A **273** 611–58
Dacie J V and Lewis S M 1970 *Practical Haematology* 4th edn (London: Churchill)
Keele C A and Neil E 1971 *Samson Wright's Applied Physiology* 12th edn (London: Oxford University Press)
Longuet-Higgins M S 1956 *Phil. Trans.* A **249** 321–87
Reif F 1965 *Fundamentals of Statistical and Thermal Physics* (New York: McGraw-Hill)
Rice S O 1944 *Bell Tech. J.* **23** 282–332
—— 1945 *Bell Tech. J.* **24** 46–156
Rschevkin S N 1963 *A Course of Lectures on the Theory of Sound* (Oxford: Pergamon)
Van de Hulst H C 1957 *Light Scattering by Small Particles* (New York: Wiley)